Recall from the first exercise session that up to isomorphism, there exist two non-isomorphic 1-dimensional real vector bundles on $S^1$, namely the trivial line bundle and the Möbius line bundle.

**Problem 1.** Let $X$ and $Y$ be topological spaces. Prove that homotopy is an equivalence relation on the set of continuous maps from $X$ to $Y$.

**Problem 2.** Let $X$, $Y$ and $Z$ be topological spaces. Prove that composition of maps descends to homotopy classes in the sense that the map

$$
\circ : [Y, Z] \times [X, Y] \longrightarrow [X, Z], \quad [f] \circ [g] = [f \circ g].
$$

is well defined. Use this to prove that homotopy equivalence is an equivalence relation among topological spaces.

**Problem 3.** Construct an explicit deformation retraction from $\mathbb{R}^n \setminus \{0\}$ to the $(n-1)$-sphere $S^{n-1} = \{x \in \mathbb{R}^n : \|x\| = 1\}$.

**Problem 4.** Show that $S^1$ and the unit 2-disk $D^2$ are not homotopy equivalent.

**Problem 5.** Which of the following letters are homotopy equivalent to which others?

\[
\begin{array}{cccccccccccccccc}
A & C & D & G & I & K & L & N & O & P & Q & R & T & X & Y
\end{array}
\]

What about the letter $B$?

**Problem 6.** Recall that a retraction from a space $X$ to a subspace $A$ is a continuous map $r : X \to A$ such that $r(x) = x$ for all $x \in A$. Prove that there does not exist a retraction from the unit 2-disk $D^2$ onto $S^1$. 