Problem 1. Show that the induced map $f^*: \tilde{K}^\leq_0(Y) \to \tilde{K}^\leq_0(X)$ only depends on the pointed homotopy class of $f: X \to Y$.

Problem 2. Let $X$ be a pointed compact Hausdorff space with basepoint $x_0$. Show that the reduced theory $\tilde{K}^\leq_0(X)$ can be recovered from the unreduced theory $K^\leq_0(X)$ as the kernel of the map

$$K^\leq_0(X) \to K^\leq_0(x_0)$$

induced by the inclusion of the basepoint into $X$.

Problem 3. Let $X$ be a pointed compact Hausdorff space. Show that

$$K^\leq_0(X) \approx \tilde{K}^\leq_0(X) \oplus K^\leq_0(pt).$$


Problem 5. Use the unreduced external product to show that $K^\leq_0(pt)$ is a graded ring (with unit). Give the definition of a graded module over a graded ring, and show that for any compact Hausdorff space $X$, the $K$-theory $K^\leq_0(X)$ is a graded module over $K^\leq_0(pt)$. 