Problem 1. Recall that for a complex vector bundle $\xi$ over $X$, we defined
\[ \lambda_t(\xi) = \sum_{k \geq 0} \lambda^k(\xi)t^k \in K(X)[[t]]. \]
Extend this definition to a map
\[ \lambda_t : K(X) \longrightarrow K(X)[[t]] \]
satisfying $\lambda_t(x + y) = \lambda_t(x)\lambda_t(y)$ for all $x, y \in K(X)$. For $x \in K(X)$, define a formal power series $\psi_t(x) \in K(X)[[t]]$ by setting
\[ \psi_{-t}(x) = -t \frac{\lambda'_t}{\lambda_t}(x). \]
Show that the coefficient of $t^k$ in $\psi_t(x)$ is $\psi_k(x)$.

Problem 2. Observe that the product $S^{2n} \times S^{2n}$ can be obtained from the wedge sum $S^{2n} \vee S^{2n}$ by attaching a $4n$-cell along a map $F : S^{4n-1} \rightarrow S^{2n} \vee S^{2n}$. Let $f$ be the composite of $F$ and the fold map
\[ \nabla : S^{2n} \vee S^{2n} \longrightarrow S^{2n} \]
which is the identity map on each wedge summand. Show that the Hopf invariant of $f$ is $\pm 2$.

Problem 3. (Five lemma). Suppose
\[
\begin{array}{ccccccc}
A_1 & \xrightarrow{f_1} & A_2 & \xrightarrow{f_2} & A_3 & \xrightarrow{f_3} & A_4 & \xrightarrow{f_4} & A_5 \\
\downarrow{\alpha_1} & & \downarrow{\alpha_2} & & \downarrow{\alpha_3} & & \downarrow{\alpha_4} & & \downarrow{\alpha_5} \\
B_1 & \xrightarrow{g_1} & B_2 & \xrightarrow{g_2} & B_3 & \xrightarrow{g_3} & B_4 & \xrightarrow{g_4} & B_5
\end{array}
\]
is a commutative diagram of abelian groups with exact rows.

(a) Assume that $\alpha_2$ and $\alpha_4$ are epimorphisms and that $\alpha_5$ is a monomorphism. Prove that $\alpha_3$ is an epimorphism.
(b) Assume that $\alpha_2$ and $\alpha_4$ are monomorphisms and that $\alpha_1$ is an epimorphism. Prove that $\alpha_3$ is a monomorphism.
(c) Conclude that if $\alpha_1$ is an epimorphism, $\alpha_2$ and $\alpha_4$ are isomorphisms, and $\alpha_5$ is a monomorphism, then $\alpha_3$ is an isomorphism.

(In typical applications, one knows that $\alpha_1$, $\alpha_2$, $\alpha_4$ and $\alpha_5$ are isomorphisms, and one uses the lemma to conclude that $\alpha_3$ is an isomorphism as well.)